Answers to written exam
at the Department of Economics winter 2018-19

Economics of the Environment, Natural Resources and Climate Change

Final exam

21 December 2018

(3 hour closed book exam)

Answers only in English

This exam question consists of 5 pages in total, including this front page.

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- Or if you otherwise violate the rules that apply to the exam

Exercise 1. Optimal climate policy in a simple growth model (indicative weight: 3/4)

Consider a simplified model of economic growth and climate change that uses the following notation:

Y = gross output before damage from climate change

g = growth rate of gross output (constant)

C = consumption of final goods

M = total use of energy

m = energy use per unit of gross output

a = share of carbon-free energy in total energy use

E = emission of CO_2

q = cost of one unit of carbon-free energy

c = cost of one unit of fossil-based energy

z = average unit cost of energy

τ = carbon tax rate

S = stock of CO_2 accumulated in the atmosphere

D = fraction of output lost due to damages from climate change

W = social welfare

ρ = rate of time preference (constant)

δ = rate of decay of carbon stock in the atmosphere (constant)

t = time

The economy is in a steady state where gross output increases at the constant rate g and where we set the initial level of output equal to 1 for convenience:

$$Y_t = e^{gt}, \qquad g > 0. \tag{1}$$

By definition, total energy use equals the energy use per unit of output multiplied by total output:

$$M_t = m_t Y_t. \tag{2}$$

Emissions of CO_2 are proportional to the total use of fossil-based energy, and we can set the proportionality factor equal to 1 by appropriate choice of units. The share of fossil-based energy in total energy use is $1 - a_t$, so total CO_2 emissions are

$$E_t = (1 - a_t) M_t, \qquad 0 \leq a_t \leq 1. \tag{3}$$

As the share a_t of fossil-free energy in total energy use increases, it becomes more and more costly to increase it even further. On the other hand, due to technical progress in green energy technologies, the unit cost of fossil-free energy decreases at the rate α for any given value of a_t. Thus we assume that the cost q_t of producing a unit of fossil-free energy is given by

$$q_t = q_0 \frac{a_t^{\eta-1}}{\eta} e^{-\alpha t}, \qquad \eta > 1, \tag{4}$$

where q_0, η and α are constants. The cost of producing a unit of fossil-based energy is c_t, and fossil energy is subject to the carbon tax τ_t, so the average unit cost of energy for firms and households is

$$z_t = a_t q_t + (1 - a_t)(c_t + \tau_t). \tag{5}$$

To keep the model simple, we abstract from capital accumulation. Hence the total consumption of final goods equals gross output minus the total cost of energy production and minus the damage cost of climate change:

$$C_t = Y_t - \overbrace{[a_t q_t + (1 - a_t) c_t] M_t}^{\text{Total cost of energy production}} - D_t Y_t. \tag{6}$$

Note that the carbon tax does not reduce consumption because the tax revenue is assumed to be recycled as a lump sum transfer to consumers. The damage cost per unit of gross output is assumed to be proportional to the accumulated stock of carbon in the atmosphere (S_t) which drives global warming:

$$D_t = \gamma S_t. \tag{7}$$

The damage cost parameter γ is treated as a constant. Over time, a constant (small) fraction $\delta > 0$ of the existing stock of CO_2 in the atmosphere is absorbed by other carbon reservoirs, but at the same time new emissions add to the carbon stock S_t which therefore evolves as

$$\dot{S}_t = E_t - \delta S_t. \tag{8}$$

This completes the description of the model.

Question 1.1. In each period firms and households choose their mix of fossil-free and fossil-based energy with the purpose of minimizing their total unit cost of energy, taking q_t, c_t and τ_t as given. Use (4) and (5) to show that the cost-minimizing share of fossil-free energy is

$$a_t = \left(\frac{c_t + \tau_t}{q_0 e^{-\alpha t}}\right)^{\varepsilon}, \qquad \varepsilon \equiv \frac{1}{\eta - 1} > 0. \tag{9}$$

Explain the economic intuition behind the result in (9). (Note that you do not need to solve an optimal control problem at this stage; you only need eqs. (4) and (5) to answer the present question).

Answer to Question 1.1: Inserting (4) in (5), we get

$$z_t = q_0 \frac{a_t^{\eta}}{\eta} e^{-\alpha t} + (1 - a_t)(c_t + \tau_t) \tag{i}$$

The first-order condition for minimization of (i) with respect to the share of carbon-free energy is

$$\frac{dz_t}{da_t} = 0 \Longrightarrow a_t^{\eta-1} q_0 e^{-\alpha t} - (c_t + \tau_t) = 0 \Longrightarrow$$

$$a_t^{\eta-1} = \frac{c_t + \tau_t}{q_0 e^{-\alpha t}}. \tag{ii}$$

Solving (ii) for a_t yields the result in (9). One way of explaining the intuition for this result is the following: The cost-minimizing mix of carbon-free and fossil energy is attained when the marginal cost of the two types of energy is the same. From (4) it follows that the cost of carbon-free energy per unit of energy consumed (denoted by TC^R) is

$$TC_t^R = a_t q_t = q_0 \frac{a_t^{\eta}}{\eta} e^{-\alpha t},$$

so the marginal cost of this type of energy (MC^R) is

$$MC^R \equiv \frac{dTC^R}{da_t} = q_0 a^{\eta-1} e^{-\alpha t}.$$

The marginal cost of fossil energy is $c_t + \tau_t$ (since c_t does not depend on a_t in the present model). Thus the optimal energy mix which equalizes the cost of carbon-free and fossil energy is given by the condition

$$MC^R = c_t + \tau_t \Longrightarrow q_0 a^{\eta-1} e^{-\alpha t} = c_t + \tau_t.$$

Solving this equation for a_t gives the result in (9). Eq. (9) has the intuitive implication that the cost-minimizing share of carbon-free energy is higher the higher the price of fossil energy and the lower the value of the factor $q_0 e^{-\alpha t}$, since a smaller value of $q_0 e^{-\alpha t}$ implies a lower cost of carbon-free energy for any given value of a_t. The factor $\eta - 1$ is the elasticity of the marginal cost of fossil-free energy with respect to the share of this type of energy. The higher the value of η, the stronger is the increase in marginal cost as the share of fossil-free energy goes up. It is therefore intuitive that the cost-minimizing value of a_t is lower, the smaller the value of $\varepsilon \equiv 1/(\eta - 1)$, i.e. the larger the value of η. (*This is the advanced explanation for the result. It is satisfactory if the student notes that it is intuitive that the cost-minimizing share of carbon-free energy is higher the higher the price of fossil energy and the lower the value of the factor* $q_0 e^{-\alpha t}$*, since a smaller value of* $q_0 e^{-\alpha t}$ *implies a lower cost of carbon-free energy for any given value of* a_t).

In the following you will be asked to characterize the optimal climate policy. For this purpose, we assume that the social planner wishes to maximize the following objective function which defines social welfare as the present value of future consumption:

$$W = \int_0^\infty C_t e^{-\rho t} dt, \qquad \rho > g. \tag{10}$$

Question 1.2. Use the relevant equations from the model above to show that the social welfare function (10) can be written as

$$W = \int_0^\infty \left\{ 1 - m_t \left[q_0 \frac{a_t^\eta}{\eta} e^{-\alpha t} + (1 - a_t)\, c_t \right] - \gamma S_t \right\} e^{-(\rho - g)t} dt. \tag{11}$$

Explain why the assumption $\rho > g$ made in (10) is important.

Answer to Question 1.2: Inserting (7) in (6) and substituting the resulting expression for C_t into (10), we find

$$W = \int_0^\infty \{ Y_t - [a_t q_t + (1 - a_t)\, c_t]\, M_t - \gamma S_t Y_t \} e^{-\rho t} dt \tag{iii}$$

Using (2) to eliminate M_t and (1) to eliminate Y_t, eq. (iii) can be rewritten as

$$\begin{aligned} W &= \int_0^\infty \{1 - m_t [a_t q_t + (1 - a_t) c_t] - \gamma S_t\} Y_t e^{-\rho t} dt \\ &= \int_0^\infty \{1 - m_t [a_t q_t + (1 - a_t) c_t] - \gamma S_t\} e^{(g-\rho)t} dt \end{aligned} \tag{iv}$$

In a last step we can insert (4) in (iv) to eliminate q_t. We then end up with (11). The assumption that $\rho > g$ (implying a positive growth-adjusted discount rate) is important since the integral in (11) will not be a finite number if this assumption does not hold.

Question 1.3. Use the relevant equations from the model above to show that the stock of carbon in the atmosphere evolves as

$$\dot{S}_t = (1 - a_t) m_t e^{gt} - \delta S_t, \qquad S_0 \text{ given.} \tag{12}$$

Answer to Question 1.3: Inserting (3) and subsequently (2) and (1) in (8) we obtain the result in (12):

$$\begin{aligned} \dot{S}_t &= E_t - \delta S_t \\ &= (1 - a_t) M_t - \delta S_t \\ &= (1 - a_t) m_t Y_t - \delta S_t \\ &= (1 - a_t) m_t e^{gt} - \delta S_t. \end{aligned}$$

Question 1.4. The optimal climate policy is the time path for the share of fossil-free energy a_t that will maximize social welfare (11) subject to (12). Set up the current-value Hamiltonian for this optimal control problem where a_t is the control variable and S_t is the state variable (denote the shadow price of S_t by λ_t).

Answer to Question 1.4: The current-value Hamiltonian is

$$H_t = 1 - m_t \left[q_0 \frac{a_t^\eta}{\eta} e^{-\alpha t} + (1 - a_t) c_t \right] - \gamma S_t + \lambda_t \left[(1 - a_t) m_t e^{gt} - \delta S_t \right]. \tag{v}$$

Question 1.5. Use the first-order condition for a_t in the optimal control problem defined in Question 1.4 to derive an expression for the socially optimal choice of the share of fossil-free energy at any given time t, written as a function of c_t and λ_t. Give an economic interpretation of this expression for a_t (note that since a higher value of S_t implies greater damage from climate change, we have $\lambda_t < 0$).

Answer to Question 1.5: From (v) we get the following first-order condition for the optimal choice of the control variable a_t:

$$\frac{\partial H_t}{\partial a_t} = 0 \Longrightarrow$$

$$-m_t \left[q_0 a_t^{\eta-1} e^{-\alpha t} - c_t + \lambda_t e^{gt} \right] = 0 \Longrightarrow$$

$$q_0 a_t^{\eta-1} e^{-\alpha t} = c_t - \lambda_t e^{gt} \Longrightarrow$$

$$a_t = \left(\frac{c_t - \lambda_t e^{gt}}{q_0 e^{-\alpha t}} \right)^{\varepsilon}, \qquad \varepsilon \equiv \frac{1}{\eta - 1} > 0. \tag{vi}$$

From (vi) we see that the optimal share of carbon-free energy is larger the higher the cost of fossil energy (c_t) and the greater the marginal damage from climate change ($-\lambda_t e^{gt} = -\lambda_t Y_t$). Since the fraction of output lost due to climate change is proportional to the concentration of CO_2 in the atmosphere (which has a negative shadow price λ_t), it is intuitive that the marginal damage from climate change $-\lambda_t Y_t$ increases in proportion to output. From (4) we see that the term $q_0 e^{-\alpha t}$ in (vi) determines the cost of fossil-free energy for any given share a_t of such energy. A higher value of $q_0 e^{-\alpha t}$ implies a higher cost of fossil-free energy for any given value of a_t. Eq. (vi) therefore implies that the optimal share of fossil-free energy is smaller the more expensive this form of energy is.

Question 1.6. Use your results in questions 1.1 and 1.5 to derive an expression for the optimal carbon tax rate τ_t as a function of the shadow price λ_t. How does the optimal carbon tax rate depend on the evolution of gross output? Explain the intuition behind your result.

Answer to Question 1.6: The optimal carbon tax rate is the value of τ_t which will ensure that the privately optimal share of fossil-free energy given by (9) equals the socially optimal share of such energy determined by (vi). We can therefore find the optimal carbon tax by setting the expressions for a_t in (9) and (vi) equal to each other and solving the resulting equation for τ_t:

$$\left(\frac{c_t + \tau_t}{q_0 e^{-\alpha t}} \right)^{\varepsilon} = \left(\frac{c_t - \lambda_t e^{gt}}{q_0 e^{-\alpha t}} \right)^{\varepsilon} \Longrightarrow$$

$$\tau_t = -\lambda_t e^{gt} > 0. \tag{vii}$$

From (8) we see that emission of an extra unit of CO_2 causes a corresponding short-run increase in the stock of CO_2 in the atmosphere. Hence it is intuitive that the optimal carbon tax rate in (vii) varies proportionally with the numerical shadow price $-\lambda_t$ of the carbon stock which reflects the marginal damage from climate change. Since $Y_t = e^{gt}$, we also see from (vii) that the optimal carbon tax rises in proportion to output. The reason is that, according to (6) and (7), the total damage cost of climate change ($D_t Y_t = \gamma S_t Y_t$) is proportional to output: The greater the volume of output, the larger the amount of output lost as carbon accumulates in the atmosphere.

Question 1.7. Go back to the optimal control problem defined in Question 1.4 and derive the first-order condition for the optimal change in the shadow price of S_t over time ($\dot{\lambda}_t$). Show that this first-order condition implies that

$$\lambda_t = -\int_t^{\infty} \gamma e^{-(\rho-g+\delta)(u-t)} du = \frac{-\gamma}{\rho - g + \delta}. \tag{13}$$

Explain the economic intuition for the result in (13). (Hint: You may use Leibniz' Rule to prove the result in (13)).

Answer to Question 1.7: From the Hamiltonian (v) we find the first-order condition for the optimal change in the shadow price λ_t to be

$$\dot{\lambda}_t = (\rho - g)\lambda_t - \frac{\partial H_t}{\partial S_t} \Longrightarrow$$

$$\dot{\lambda}_t = (\rho - g + \delta)\lambda_t + \gamma. \tag{viii}$$

Integrating (viii), we obtain eq. (13). To prove that (13) does indeed follow from (viii), we can use Leibniz's Rule which says that a function of the form

$$\lambda(t) = \int_{z(t)}^{v(t)} f(t,u)\, du$$

has the derivative

$$\lambda'(t) \equiv \dot{\lambda}_t = f(t, v(t))\, v'(t) - f(t, z(t))\, z'(t) + \int_{z(t)}^{v(t)} \frac{\partial f(t,u)}{\partial t} du \tag{ix}$$

In the present case we have from (13),

$$\lambda(t) = -\int_t^\infty \gamma e^{-(\rho-g+\delta)(u-t)} du$$

so

$$f(t,u) = -\gamma e^{-(\rho-g+\delta)(u-t)}, \qquad z(t) = t, \qquad v(t) = \infty.$$

From this it follows that

$$z'(t) = 1, \qquad v'(t) = 0, \qquad \frac{\partial f(t,u)}{\partial t} = -(\rho-g+\delta)\gamma e^{-(\rho-g+\delta)(u-t)}. \tag{x}$$

which may be inserted into formula (ix) to give

$$\begin{aligned} \lambda'(t) &\equiv \dot{\lambda}_t = \gamma - \int_t^\infty (\rho-g+\delta)\gamma e^{-(\rho-g+\delta)(u-t)} du \\ &= \gamma - (\rho-g+\delta) \overbrace{\int_t^\infty \gamma e^{-(\rho-g+\delta)(u-t)} du}^{=-\lambda(t)} \\ &= \gamma + (\rho-g+\delta)\lambda(t). \end{aligned} \tag{xi}$$

We see that the last line in (xi) is identical to the right-hand side of (viii). This proves that the first-order condition (viii) does indeed imply the expression for the shadow price λ_t stated in (13).

According to (7) and (13) the (numerical) shadow price of the carbon stock S_t equals the present value of the future damage costs per unit of output generated by a unit increase in the current carbon stock. If the carbon stock increases by one unit at time t, the resulting increase in the carbon stock at the future time $u > t$ will be $e^{-\delta(u-t)}$, since the stock of carbon in the atmosphere decays at the exponential rate δ (this follows from (8)). From (7) it therefore follows that a unit increase in S at time t will cause a total damage at time u equal to $\gamma e^{-\delta(u-t)} Y_u$. From (1) we have $Y_u = Y_t e^{g(u-t)}$. Hence the damage per unit of output at time u caused by a unit increase in S at time t will be $\gamma e^{(g-\delta)(u-t)}$. This cost has a present value at time t equal to $\gamma e^{-(\rho-g+\delta)(u-t)}$, since future costs are dicounted at the rate ρ. These observations provide the intuition for the expression for the shadow price λ_t given in (13).

(Note: Students are not expected to provide an explanation for eq. (13) which is quite as elaborate as the one given above. Furthermore, Question 1.7 is difficult, as many

students will not be able to remember Leibniz' Rule, so only the best students will be able to answer this question in a fully satisfactory manner).

Exercise 2. The debate on Integrated Assessment Models (indicative weight: 1/4).

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful).

Question 2.1: Describe (briefly) the main features of a typical Integrated Assessment Model of the economy and the climate system such as the DICE model.

Answer to Question 2.1: An Integrated Assessment Model (IAM) combines a description of the economic system with a simplified description of the climate system and of the interaction between the two systems. In the DICE model developed by William Nordhaus the world economy is described by a Ramsey-type one-sector model of economic growth driven by endogenous capital accumulation and exogenous growth in population and total factor productivity.

Production causes emissions of CO_2, albeit at a declining rate per unit of output due to exogenous improvements in energy efficiency and abatement technologies. The CO_2 emissions feed into the global carbon cycle where carbon circulates between the atmosphere, the upper oceans and the lower oceans. The net result of the anthropogenic additions to the carbon cycle is a gradual increase in the concentration of CO_2 in the atmosphere causing a gradual increase in the average mean global temperature at the surface of the Earth via radiative forcing (the difference between the amount of sunlight energy the Earth receives from the sun and the amount of energy radiated back into space). Thus the DICE model includes a simplified modelling of the greenhouse effect that causes global warming by hampering the radiation of sunlight energy back into space.

Global warming and the resulting climate change leads to output losses which are modelled as a non-linear function of the average global temperature. CO_2 emissions can be abated by spending a part of total output on abatement effort. The abatement cost function in the DICE model implies that the marginal abatement cost is increasing in the level of abatement. Social welfare is measured by the discounted value of the total future utility of the world population. The utility of the representative consumer is assumed to depend on his/her level of consumption and the utility function implies decreasing marginal utility of consumption. When calibrated with plausible parameter values derived from economic research and climate science, the DICE model can in principle be used to determine the optimal climate policy that maximizes social welfare. The optimal policy is attained when the marginal abatement cost in each period is equal to the Social Cost of Carbon (SCC) in that period. The SCC in period t is defined as the present value of all the future damage costs caused by the emission of an extra ton of CO_2 in period t. Since cost-minimizing economic agents will abate their CO_2 emissions up to the point where their marginal abatement cost equals their private cost of emitting an extra ton of CO_2, the optimal climate policy can in principle be implemented via a carbon tax which is set equal to the SCC (or via a system of tradable emission allowances that establishes an allowance price equal to the SCC).

Question 2.2: Discuss some strengths and weaknesses of the DICE model.

Answer to Question 2.2: The strength of Integrated Assessment Models is that they describe the interaction between the economy and the climate system. Hence they do not only describe the effects of economic growth on the climate but also how climate change feeds back to the economy via damages to the economic system. Thus an IAM like the DICE model can not only be used to estimate the abatement effort needed to attain a given target for global warming; it can also be used to calculate the optimal abatement effort that balances the marginal benefit and cost of abatement.

However, standard Integrated Assessment Models have met with several criticisms, including the following points:

1) The optimal climate policy in IAMs depends crucially on the discount rate about which there is great ethical controversy and great uncertainty in the far future.

2) Standard IAMs model damages from climate change as a reduction in conventional consumption possibilities. They thereby implicitly make the restrictive assumption that conventional goods and environmental goods are perfect substitutes. In practice climate change is likely to increase the scarcity value of environmental goods relative to the value of conventional goods.

3) The empirical basis for estimating the damage function in IAMs is very weak since

global warming above 3 °C has not been seen on the planet for around 3 million years. The properties of the damage function above this level of warming is pure guesswork.
4) Standard IAMs do not adequately capture the risk of catastrophic climate change that may occur beyond highly uncertain "tipping points".
5) Standard IAMs assume implausibly that the growth of total factor productivity is unaffected by climate change.

All of these points can be elaborated in various ways. For example, a discussion of point 1) could start from the Ramsey formula for the consumption discount rate (r) which is

$$r = \rho + \sigma g, \tag{xii}$$

where ρ is the rate of time preference (the rate at which future utility is discounted in the social welfare function), g is the growth rate of consumption, and σ is the elasticity of the marginal utility of consumption which indicates how fast the marginal utility declines when consumption increases. In the big debate on discounting, Nicholas Stern has argued that the choice of the values of ρ and σ is an ethical one that reflects society's preferences regarding the distribution of welfare. Specifically, ρ reflects how society weighs the welfare of current generations against the welfare of future generations, and σ reflects how society weighs the consumption of poor persons against the consumption of rich persons.

However, time will not allow students to provide extensive explanations of all the five points mentioned above.